

Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year		Final Term Exam Course: Mathematics 1 – A Date: January 5 , 2019 Duration: 3 hours
The Exam consists of one page Answer All Questions No. of questions: 4 Total Mark: 100		
Question 1		
(a)Complete the following :		5
(i)The sum of all eigenvalues of a unit matrix of order n equals to.....		
(ii)A square matrix A is called non singular ifand it is called symmetric if		
(iii)If λ is eigenvalue of a matrix A , then $\lambda + \sqrt{\lambda}$ is eigenvalue to		
(iv)A linear system $AX = B$ is called inconsistent if		
(v)A linear system $AX = B$ has one solution if		
(b)If $A = \begin{bmatrix} 0 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -3 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$		8
Find, if possible A + B , A + C, A.B , A.C , C.A , A , C		
(c)Find the eigenvalues and the eigenvectors of the matrix : $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$		8
Also, Write the diagonal form and find $f(A) = \ln A$.		
(d)Solve the L.S : (i) $x - 2y + z = 1$, $-2x + 3y - z = 0$, $3x + y - 2z = -3$		2
(ii) $x - y + 2z = 2$, $-x + y - z = 3$, $3x - 3y + 4z = 3$		2
Question 2		
(a)Find the coefficient of y^4 in the expansion of $(y^2 + \frac{1}{y^2})^6$	(b)Expand $\frac{2x-3}{(x-1)(x-2)}$	3+3
(c)If $z_1 = 2 + 2i$, $z_2 = 8i$. Find $(z_1.z_2)^6$, $(\frac{z_2}{z_1})^{10}$ and $\sqrt[4]{z_2}$.		4
(d)Find u and v of the function $f(z) = \cos(iz)$.		2
(e)Solve $4x^3 - 24x^2 + 48x - 32 = 0$ if its roots a, b, c form a geometric sequence.		3
(f)Find S_n , S_{100} and S_∞ for the series : $\sum_{r=1}^n \frac{1}{(3r-1)(3r+2)}$		5
(g)By Induction, prove that : $9^n - 2^n$ is divisible by 7, $n \geq 1$.		5
Question 3		
(a)Find $\frac{dy}{dx}$ where : (i) $y = 3\sin^3 t$, $x = 2\cos^3 t$	(ii) $e^x \sin y = e^y \cos x$	12
(iii) $y = e^{5x^2} . (\operatorname{arcsec} x)^8 . (7 - \operatorname{csch} 3x)^{-12}$		
(b)Find $y^{(18)}$ where $y = \frac{3-5x}{6x+8}$.		5
(c)Find the integral : $\int (8^{3x} + x^2 e^{-5x^3} - \cot 4x + \frac{x^3}{\sqrt{x-5}})dx$		8
Question 4		
(a)If the hypotenuse of right triangle is constant. Show that its area is maximum when it is isosceles .		5
(b)Find the equation of tangent and normal for the curve: $y = \frac{(\ln x)^x}{2^{\ln x - 1}}$ at $x = e$.		8
(c)Find the limits: (i) $\lim_{x \rightarrow \infty} \frac{5^x + 2^x}{9^x}$ (ii) $\lim_{x \rightarrow \infty} x^{\frac{1}{\sqrt{x}}}$ (iii) $\lim_{x \rightarrow \infty} e^{-x} \ln x$ (ii) $\lim_{x \rightarrow 0} (\frac{1}{x} - \frac{1}{e^x - 1})$		12
Good Luck	Dr. Khaled Elnaqar	Dr. Mohamed Eid
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Model Answer

Answer of Question 1

- (a)(i) The sum of all eigenvalues of a unit matrix of order n equals to n .
(ii) A square matrix A is called non singular if $|A| \neq 0$ if and it is called symmetric if $A = A'$.
(iii) If λ is eigenvalue of a matrix A , then $\lambda + \sqrt{\lambda}$ is eigenvalue to $A + \sqrt{A}$.
(iv) A linear system $AX = B$ is called inconsistent if it has no solution.
(v) A linear system $AX = B$ has one solution if $\text{rank } A = \text{rank } G = \text{number of unknowns}$.

-----5 Marks

(b) If $A = \begin{bmatrix} 0 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -3 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$

$A + C$, $A.B$, $C.A$ and $|A|$ are not exist.

$$A + B = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 0 & 0 \end{bmatrix}, \quad A.C = \begin{bmatrix} 4 & -4 & 10 \\ 7 & -2 & 6 \end{bmatrix}, \quad |C| = 1 + 8 - 6 = 3$$

-----8 Marks

(c) $\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda) - 2 = \lambda^2 - 5\lambda + 4 = 0$. Then $\lambda_1 = 1$, $\lambda_2 = 4$

From the equation, $\begin{bmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

For: $\lambda_1 = 1$, $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then $x + 2y = 0$, $x + 2y = 0$

Put $x = 2$, then the corresponding

eigenvector is: $X_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

For: $\lambda_2 = 4$, $\begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then $-2x + 2y = 0$, $x - y = 0$

Put $y = 1$, we get $x = 1$ and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Then $T = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$ and $T^{-1} = \frac{1}{3} \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

The diagonal form: $T^{-1}AT = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$. Then $T^{-1} \ln A T = \begin{bmatrix} \ln 1 & 0 \\ 0 & \ln 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1.4 \end{bmatrix}$

Then $\ln A = T \begin{bmatrix} 0 & 0 \\ 0 & 1.4 \end{bmatrix} T^{-1} = \frac{1}{3} \begin{bmatrix} 1.4 & 2.8 \\ 1.4 & 2.8 \end{bmatrix}$

-----8 Marks

(d) By Calculator, the solution is: (1, 2, 4).

(ii) $G = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ -1 & 1 & -1 & 3 \\ 3 & -3 & 4 & 3 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & -2 & -3 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 7 \end{array} \right]$

Rank $A = 2$, Rank $G = 3$. No solution.

-----4 Marks

Answer of Question 4

a) Since the hypotenuse of the right triangle is constant, therefore $x^2 + y^2 = c^2 \Rightarrow 2x + 2yy' = 0 \Rightarrow y' = -x/y$

$$\text{Area: } A = \frac{1}{2}xy \Rightarrow \frac{dA}{dx} = \frac{1}{2}(xy' + y) = \frac{1}{2}\left(x\left(-\frac{x}{y}\right) + y\right) = 0 \Rightarrow x^2 = y^2 \Rightarrow x = y$$

Therefore the area is maximum when the triangle is isosceles.

$$\text{b) } y' = \frac{[\ln x]^x [\ln(\ln x) + x(\frac{1}{x})][2^{\ln(x)-1}] - [\ln x]^x [\frac{1}{x}][2^{\ln(x)-1}] \ln 2}{[2^{\ln(x)-1}]^2}$$

$$\text{at } x = e, \text{ slope of the tangent} = y'(1) = 1 - \frac{\ln 2}{e} = \frac{e - \ln 2}{e}$$

Therefore the point of contact is (e, 1) and hence the equation of tangent is $\frac{y-1}{x-e} = \frac{e - \ln 2}{e}$

$$\text{Also equation of normal is } \frac{y-1}{x-e} = \frac{e}{\ln 2 - e}$$

$$\text{c-i) } \lim_{x \rightarrow \infty} \frac{5^x + 2^x}{9^x} = \lim_{x \rightarrow \infty} \frac{5^x(1 + (\frac{2}{5})^x)}{9^x} = \lim_{x \rightarrow \infty} \left(\frac{5}{9}\right)^x = 0$$

$$\text{ii) This limit of the indeterminate form } \infty^0. \text{ Let } y = x^{1/\sqrt{x}} \Rightarrow \ln y = \frac{\ln x}{\sqrt{x}}$$

Hence L'Hopital rule is ready to be used such that

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{1/x}{1/(2\sqrt{x})} = \lim_{x \rightarrow \infty} \frac{2}{\sqrt{x}} = 0.$$

$$\text{Therefore } \lim_{x \rightarrow \infty} x^{1/\sqrt{x}} = \lim_{x \rightarrow \infty} e^{\ln y} = e^{\lim_{x \rightarrow \infty} \ln y} = e^0 = 1$$

$$\text{iii) } \lim_{x \rightarrow 0} \left[\frac{1}{x} - \frac{1}{e^x - 1}\right] = \lim_{x \rightarrow 0} \left[\frac{e^x - 1 - x}{x(e^x - 1)}\right] = \lim_{x \rightarrow 0} \left[\frac{e^x - 1}{xe^x + (e^x - 1)}\right] = \lim_{x \rightarrow 0} \left[\frac{e^x}{xe^x + 2e^x}\right] = \frac{1}{2}$$

$$\text{iv) } \lim_{x \rightarrow \infty} e^{-x} \ln x = \lim_{x \rightarrow \infty} \frac{\ln x}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{xe^x} = 0$$

Dr. Khaled El Nagar